NUMERICAL MODELING OF HEAT AND MASS TRANSFER IN FIBER BUNDLES WITH LIQUID EVAPORATION ON THEIR SURFACES

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In a number of technological processes for the spinning and processing of synthetic fibers, heat and mass transfer are accompanied by the liberation of gas from the surface of the fibers. As a rule, it is induced by chemical reactions such as combustion, by decomposition of the polymer with the precipitation of low-molecular compounds, or by liquid evaporation. In all these cases heat and mass transfer are influenced by the heat of reaction or the heat of phase transition. Heat transfer with phase transition of the liquid is an important process in the drying of fibrous materials and fiber bundles.

Of all the many and varied existing technological operations, the drying of moving fiber bundles can be regarded in isolated cases as a certain peripheral link in the greater technological chain, but it is still of practical interest. It can involve such stages as extraction of the fiber bundle from the settling tank after wet spinning and the motion of the bundles after lubrication.

The drying of moving fiber bundles plays an equally major role in dry spinning, where the polymer solvent is evaporated from the fiber surfaces. Since the fiber contains considerably more solvent than polymer in the initial stage, the evaporation of the liquid takes place qualitatively in accordance with the laws governing the drying of moisture-impregnated porous materials [1-3]. This type of process is characterized by several stages, the first of which is accompanied by variation of the fiber temperature and rapid evaporation, while vigorous gas liberation is maintained in the second stage, with equal heat fluxes proceeding from the medium and entering into phase transition, so that the temperature of the filaments remains practically constant. Cross-linking of the polymer and the development of fiber properties take place simultaneously with the removal of the solvent. In the light of this, the development of mathematical models of combined heat and mass transfer during phase transitions in moving fiber bundles and the application of those models in computations of real spinning setups pose a timely problem.

1. STATEMENT OF THE PROBLEM

We assume that each fiber in the bundle is coated with a thin film of water, whose thickness is specified. We consider the accompanying evaporation to be an equilibrium process, i.e., the saturation concentration in the gas near the surface of the film is related one-to-one to the pressure and temperature in this zone. We also assume that the velocity of the film and the temperature of the liquid are equal to the velocity and temperature of the fiber. The equations of motion and heat transfer of a bundle of moving fibers or filaments have been derived [4] on the basis of a filtration flow model and boundary-layer model. This system of equations has been augmented [5] with diffusion equations, but mass and heat transfer are treated separately in the cited paper. In combined heat- and mass-transfer studies these equations must be further elaborated with terms to account for heat transfer as a result of the diffusion motion of the components. The majority of gas mixtures comply approximately with the condition $Le = Pr/Sc = Dc_p/\lambda \approx 1$ (Le is the Lewis–Semenov number, Pr is the Prandtl number, and Sc is the Schmidt number), and we shall assume that this condition holds exactly in the equations for our case. Since the heat and mass fluxes are given by the following functional relations for a two-component without regard for thermal and pressure diffusion [6, 7]:

$$q = -\lambda \operatorname{grad} T + \sum_{i=1}^{\iota} h_i g_i,$$

$$g_i = \rho c_i V_i = \rho c_i v - D c_i^{-1} \operatorname{grad} c_i, \tag{1.1}$$

the basic equations of motion and heat and mass transfer for an incompressible medium in bundles of rods of constant diameter have the form

$$\varepsilon^{-1} \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} \right) = -\varepsilon \frac{dp}{dx} + R_x - \varepsilon^{-1} \rho u G$$

$$+ \frac{1}{r} \frac{\partial}{\partial r} \left(\mu r \frac{\partial u}{\partial r} \right),$$

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = rG,$$

$$\varepsilon^{-1} \rho \left(u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial r} \right) = \varepsilon^{-1} Q - \varepsilon^{-1} \rho h G \tag{1.2}$$

$$+ \frac{1}{r} \frac{\partial}{\partial r} \left(\rho D r \frac{\partial h}{\partial r} \right),$$

$$\varepsilon^{-1} \rho \left(u \frac{\partial c_1}{\partial x} + v \frac{\partial c_1}{\partial r} \right) = \varepsilon^{-1} K - \varepsilon^{-1} \rho c_1 G + \frac{1}{r} \frac{\partial}{\partial r} \left(\rho D \frac{\partial c_1}{\partial r} \right).$$

Here x, r is the coordinate system; u and v are the components of the filtration velocity; h is the enthalpy; h_i is the enthalpy of the ith component; ρ is the density; p is the pressure; ε is the porosity of the bundle; c_p is the specific heat at constant pressure; μ is the dynamic viscosity coefficient; λ is the thermal conductivity; c_i is the mass concentration of the ith component; V_i is the velocity of molecules of the ith component; v is the real "bulk" velocity (mass flow rate averaged over the flow cross section); D is the diffusion coefficient. The subscript 1 associates the parameters with the vapor, and the subscript 2 associates them with air. The sources G, R_x, Q, and K determine the interaction of the system of moving fibers with the surrounding medium. According to [5], they can be represented as follows in connection with Eq. (1.1):

$$G = \frac{2\pi R_\text{в}}{\pi R_\Delta^2} v_\text{в}, \quad R_x = \frac{2\pi R_\text{в}}{\pi R_\Delta^2} (\rho U_\text{в} v_\text{в} - \tau_\text{в}),$$

$$Q = \frac{2\pi R_\text{в}}{\pi R_\Delta^2} \left(\rho v_\text{в} h_\text{в} - q_\text{в} - \sum_{i=1}^{2} h_\text{вi} g_\text{вi} \right), \tag{1.3}$$

$$K = \frac{2\pi R_\text{в}}{\pi R_\Delta^2} (\rho c_{1\text{в}} v_\text{в} - g_\text{в}),$$

where R_B is the radius of the fiber plus the film; R_Δ is the radius of the cell; U_B is the velocity of the fiber; v_B is the hydrodynamic flow velocity from the film surface during evaporation; h_B and $c_{1\text{B}}$ are the enthalpy of the mixture and the saturated vapor concentration on the film surface; and τ_B; q_B; and g_B are the frictional stress and the thermal and diffusion fluxes on the film surface. The expression for Q differs from the one obtained in [5] in that the enthalpy rather than the temperature of the mixture is used in calculating the energy transfer in a two-component medium. In this case the solution for the enthalpy of the gas moving in the cell with the liberation of gas from the fiber surface has the following form by analogy with Eq. (2.4) in [5]:

$$h = h_\text{в} + (h_\Delta - h_\text{в}) f_0 (r, \alpha_\text{c}). \tag{1.4}$$

Differentiating (1.4) with respect to the radius at $r = R_\text{B}$, according to [7], we find

$$\frac{\partial h}{\partial r} \bigg|_{r=R_\text{в}} = c_p \frac{\partial T}{\partial r} \bigg|_{r=R_\text{в}} + \sum_{i=1}^{2} h_\text{вi} \frac{\partial c_i}{\partial r} \bigg|_{r=R_\text{в}}$$

$$= c_p \frac{\partial T}{\partial r} \bigg|_{r=R_\text{в}} - \sum_{i=1}^{2} \frac{h_\text{вi}}{D} g_\text{вi}.$$
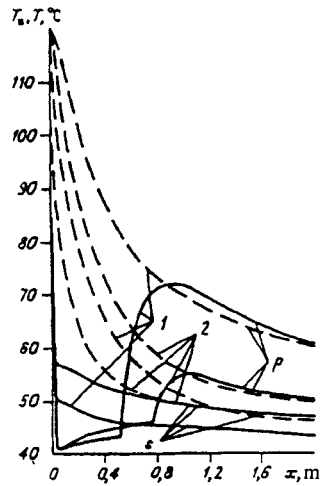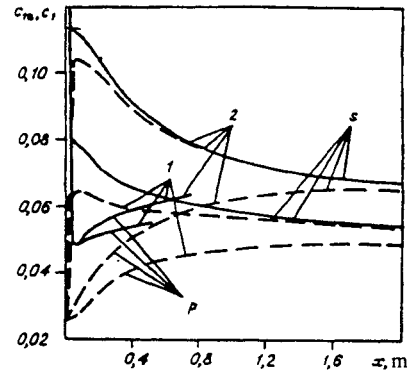
Fig. 1

Fig. 2

Allowing for the fact that $q_B = -\lambda \partial T/\partial r|_{r=R_B}$, we have

$$q_s = -\frac{\lambda}{c_p}\frac{\partial h}{\partial r}\bigg|_{r=R_s} - \sum_{i=1}^{2} Le^{-1} h_{si} g_{si},$$

whereupon we finally obtain

$$Q = \frac{2\pi R_s}{\pi R_\Delta^2}\left(\rho v_s h_s - \frac{\lambda}{c_p}\frac{\partial h}{\partial r}\bigg|_{r=R_s}\right),$$

i.e., the source in the energy equation does not depend explicitly on the enthalpy or the mass fluxes of the components of the mixture. The procedure for determining τ_B, g_B, and $\partial h/\partial r|_{R_B}$ is the same as that described in [5] and is omitted here. The hydrodynamic velocity during evaporation is determined from the relation [1]

$$v_s = -\frac{D}{1-c_{1s}}\frac{\partial c_1}{\partial r}\bigg|_{r=R_s}. \tag{1.5}$$

The saturation concentration c_{1B} on the surface of the evaporating liquid, subject to the condition that the process is in equilibrium, is determined from relations between the temperature and pressure of the saturated vapor. For fibers of radius 0.01 mm or more we can disregard σ_L/R_B in comparison with atmospheric pressure and assume that the vapor pressure over the cylindrical liquid film is equal to the pressure over a flat surface (σ_L is the coefficient of surface temperature).

We augment the system (1.2)–(1.5) with the heat-transfer equation of the elementary fiber; in the approximation of a thermally slender body with allowance for the heat of phase transition this equation has the form

$$\pi R_s^2 \rho_\Sigma c_{p\Sigma} U_s \frac{dT_s}{dx} = -2\pi R_s(q_s + \rho v_s H), \tag{1.6}$$

where $\rho_\Sigma c_{p\Sigma} = \beta\rho_B c_{pB} + (1-\beta)\rho_L c_{pL}$, $\beta = R_*^2/R_B^2$, R_* is the radius of the bare (without the film) fiber; H is the specific heat of vaporization; ρ_B and c_{pB} are the density and specific heat of the polymer, ρ_L and c_{pL} are the density and specific heat of the liquid.

Equations (1.2)–(1.6) with the appropriate initial and boundary conditions describe conjugate heat and mass transfer in a bundle of moving fibers with liquid evaporating from their surfaces. However, one more assumption is made in our calculations. Allowing for the fact that the specific heat c_p of the mixture varies as a result of the slight variation of the concentrations, we assume that $h = c_p T$. This enables us to change from enthalpy to temperature in the heat-transfer equation. The specific temperature dependences of the diffusion coefficient, pressure, saturated vapor concentration, and heat of vaporization determine the form of the evaporating liquid.

As an example, we consider the equilibrium water evaporation process. According to [8], the saturated vapor pressure approximation in the temperature range from 0° to 320°C is written for this process in the form
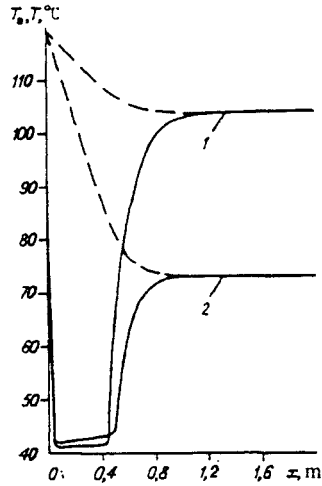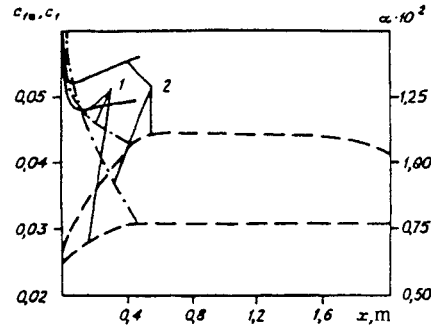
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Fig. 3

Fig. 4

$$p_{1s} = 10^6 \left\{ \exp \left[\sum_{i=-1}^{1} a_i \left(\frac{T_s}{1000} \right)^i + a_2 \ln(T_s) \right] \right\} \qquad (1.7)$$

($a_{-1} = -7.8215$, $a_0 = 82.8657$, $a_1 = 10.2100$, and $a_2 = -11.4878$).

The saturated vapor concentration as a function of the pressure is determined from the relation [9]

$$c_{1s} = M_1 y \left[M_1 y + M_2 (1 - y) \right]^{-1}, \quad y = p_{1s}/\Pi, \qquad (1.8)$$

where M_1 and M_2 are the molar masses of the vapor and air, respectively, and Π is the pressure of the air—vapor mixture ($\Pi = p$).

The diffusion coefficient of water vapor in air has the form [8]

$$D = D_0 (T/T_0)^{1.8} p_0 / p \qquad (1.9)$$

($D_0 = 0.216$ m^2/s at $p_0 = 1.01 \cdot 10^5$ Pa and $T_0 = 273$ K).

Tabulated data [10] are used to approximate the temperature dependence of the dynamic viscosity of water vapor. The dynamic viscosity of air is determined from Sutherland's formula [7]. The viscosity of the mixture depends on the vapor viscosity μ_1, the air viscosity μ_2, and the concentrations of the components and is determined from the equation

$$\mu = \frac{\mu_1 c_1}{c_1 + (1 - c_1)\psi_{12}} + \frac{\mu_2(1 - c_1)}{1 - c_1 + c_1 \psi_{21}}, \qquad (1.10)$$

where ψ_{12} and ψ_{21} are correction factors, which depend on the molecular weights and viscosities of the components [8].

The specific heat of vaporization of water is calculated from the approximation relation [9]

$$H = 10^3 \exp(-0{,}000989 T_s \, {}^\circ C + 7{,}8213). \qquad (1.11)$$

The system consisting of Eqs. (1.2)–(1.11) must be augmented with transport equations in the exterior region surrounding the fiber bundle. They can be obtained from (1.2) by setting $\varepsilon = 1$, $G = R_x = Q = K = 0$. Interaction between the flow regions is taken into account by matching conditions at the boundary of the bundle for the velocities, temperatures, concentrations, tangential stresses, and heat and mass fluxes.
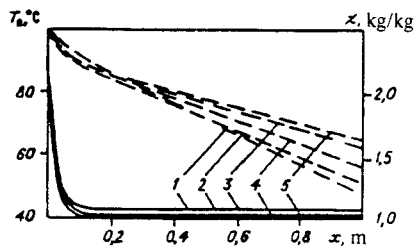
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Fig. 5

2. RESULTS OF CALCULATIONS

We investigate model heat- and mass-transfer problems for a bundle of fibers in an open medium and in a pot (blowdown line); these models correspond to the principal setups used for the spinning of complex fibers. We assume that the bundle has a radius $R_b = 0.025$ m and is made up of fibers with an initial radius $R_{B0} = 0.7 \cdot 10^{-4}$ m coated with a water film of thickness $0.1R_{B0}$. The fibers move with a velocity $U_B = 0.35$ m/s and have an initial temperature $T_{B0} = 85°C$. The ambient temperature is $T_\infty = 120°C$, and the water vapor concentration is $c_{1\infty} = 0.025$; here Pr $= 0.7$ and Sc $= 0.7$. The following boundary conditions are stipulated at infinity for the system of equations (1.2) in the open-air spinning model: $u(\infty) = 0$, $T_B(\infty) = T_\infty$, $c_1(\infty) = c_{1\infty}$.

Figures 1 and 2 show the numerically determined distributions of the temperatures of the fibers T_B (Fig. 1, solid curves) and the gas T (Fig. 1, dashed curves), the saturated concentrations c_{1B} (Fig. 2, solid curves), and the water vapor concentrations c_1 (Fig. 2, dashed curves) at the center of the bundle (curves s) and on its surface (curves p). The calculations refer to two bundles with N = 20 and 60 fibers (curves 1 and 2, respectively). The values of the thermophysical parameters of the polymer corresponded to polyethylene terephthalate and are taken from [11]. It follows from these distributions that the temperature of the fibers drops during the very rapid evaporation in the initial period, and then it changes more slowly in accordance with the local heat- and mass-transfer conditions. A slight temperature increase is observed on the part of the outer filaments in connection with the entry of hot gas from the outside into the bundle and a simultaneous decrease in the evaporation rate due to the decrease in the difference between c_{1B} and c_1. The inner filaments are characterized by a continuing gradual drop in temperature with a simultaneous decrease in the evaporation rate. The temperatures of the gas vary monotonically as they approach the corresponding fiber temperatures. Figure 2 shows the distributions of the saturation concentrations c_{1B} corresponding to the fiber temperatures T_B, along with the distributions of the concentrations in the vapor and the gas. Here the vapor concentration is observed to tend uniformly to the saturation level, and the value of c_1 becomes essentially equal to c_{1B} in the interior regions of the bundle. Cooling of the outer filaments causes the corresponding saturation concentrations to drop very abruptly, but then they settle into certain stabilized values. The subsequent variation of these parameters is not shown, because the water film has evaporated completely from the fibers. After the completion of evaporation the temperatures of the fiber surfaces are observed to rise sharply to the corresponding temperatures of the gas; this phenomenon is particularly visible for N = 20 (Fig. 1). It is followed downstream by an interval in which the temperature of the gas continues to drop as a result of heat being withdrawn by the inner layers of the bundle, and the fibers begin to cool.

These distributions clearly exhibit the influence of the density of the bundle on heat and mass transfer. For N = 60 the heat-transfer interval up to complete evaporation of the film from the outer filaments is much longer, and the postevaporation heating of the fibers is significantly weaker.

Curves representing the temperatures and concentrations of vapor in a bundle moving through a blowdown line are shown in Figs. 3 and 4. The boundary conditions are stipulated as follows:

$$u(R_T) = 0, \quad T(R_T) = T_w, \quad \partial c / \partial r\big|_{r=R_T} = 0$$

(R_T is the radius of the tube). The parameters of the blowdown line are $R_T = 0.075$ m and $T_w = 120°C$, and the mass flow of injected air is $G_0 = 0.025$ m^3/s. All other parameters of the problem and notation in Figs. 3 and 4 are the same as in the previous heat-transfer model. Since almost-uniform thickness profiles of the velocities, temperatures, and concentrations are observed in the bundle for the indicated parameters of the problem, Figs. 3 and 4 show only the values corresponding to the

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central regions of the bundle. A comparison of the temperature and concentration distributions in Figs. 1 and 2 with those in Figs. 3 and 4 reveals that they differ only slightly. However, the following distinctions should be noted: After a drop in the initial interval the saturation concentration c_{1B} profiles tend almost immediately to constant values corresponding to the fiber temperatures, and the distributions of the vapor and air concentrations approach c_{1B} linearly. Prior to complete evaporation of the water film the fiber temperatures depend on the number of filaments in the bundle. After the completion of evaporation, as in the open-air bundle, sudden heating of the fibers is observed, and their temperatures approach the air temperature; however, unlike the open-air case, where complete evaporation of the films takes place only in a shallow surface zone of the bundle, water evaporation from the filaments in a blowdown line is observed throughout the entire volume of the bundle.

The dot-dash curves in Fig. 4 represent the distributions of the dimensionless evaporation rates $\alpha = v_B R_B / \nu$ for N = 20 and N = 60, which illustrate the influence of the bundle packing density on the fiber drying process.

To determine the quantitative differences between the heat- and mass-transfer laws in bundles of moving filaments and in solitary fibers, we perform calculations of the drying process in the dry spinning of fibers from polyvinyl alcohol using a set of data corresponding to [3]: initial fiber velocity 0.7 m/s; initial radius $0.5 \cdot 10^{-4}$ m; air temperature 120°C; initial fiber temperature 85°C; number of fibers N = 10. The thermophysical parameters of the polymer are taken from [12]. The results of the calculations are shown in Fig. 5. For solitary fibers we consider the evaporation of water in laminar and turbulent flow, assuming similarity of the heat- and mass-transfer processes. The following expression is used as a dimensionless relation for the Nusselt number in turbulent flow [2]:

$$Nu = 0,42 Re_D^{0,333},$$

where $Nu = \alpha_B D_B / \lambda$, $Re_D = U_B D_B / \nu$, α_B is the heat-transfer coefficient, and D_B is the fiber diameter. The resulting distributions of the temperature T_B (solid curves) and moisture content χ (dashed curves) of the fiber are represented by curves 2 in Fig. 5; curves 1 give the experimental data. The agreement of these distributions is perfectly satisfactory, and the fiber temperatures practically coincide in the scale of the figure. Curves 3 represent the results of calculations for laminar flow with the dimensionless heat-transfer number specified in the form [2]

$$Nu = 4,3(a + a^2 - a^3) \ (a = [\ln \xi^2 / 4]^{-1}, \ \xi = 8 \ [x/(Re_D D_s)]).$$

The temperature difference between laminar and turbulent flow is not very great and actually occurs only in the initial interval. The distribution of the moisture content in this case differs appreciably from the experimental results because of the decreases in the heat-transfer and evaporation rates.

The results of heat- and mass-transfer calculations for a fiber bundle are represented by curves 4 (motion in a blowdown line) and 5 (open-air bundle, surface filaments). The radius of the bundle is 0.025 m, and the average velocity of the gas in the line is $U_{av} = 4U_B$. The temperatures of the fibers moving in the blowdown line and in open air are somewhat lower than the experimental data. The distributions of the moisture content for these models differ considerably from those obtained for the open-air bundle. The disagreement is attributable to the fact that the rate of evaporation of liquid on the fibers is significantly decreased by dynamical, thermal, and diffusion interaction of the fibers and the formation of a boundary layer on the bundle. The bundle in the blowdown line is characterized by forced blowing of the filaments. Its intensity depends on the injected air rate and can fluctuate over a wide range, determining the rate of change of the moisture content. In the given situation the moisture-content curve is situated between the distributions for solitary fibers in the turbulent and laminar regimes. A decrease in U_{av} brings the calculated results closer to the moisture-content distribution for a solitary fiber in laminar flow. The reported data exhibit the principal laws of drying of fiber bundles in dry spinning and show that the postulated mathematical model can be used to analyze heat- and mass-transfer processes in real technological fiber producton setups.

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